Soundness and Completeness of Symmetric Relators

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Abstract

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Compositionality. Given a labeled transition systems $(S, L, \to)$, where S is a set of states, L is a set of labels, and $\to \subseteq S \times L \times S$ is a set of labeled transitions, a *simulation* is a relation $r \subseteq S \times S$ such that the following sentence holds:

$$(x, y) \in r, x \xrightarrow{l} x' \Rightarrow \exists y', y \xrightarrow{l} y' \text{ and } (x', y') \in r$$

The greatest simulation relation over S is called *similarity* and shown with $\leq$. A simulation relation r on S is a *bisimulation* iff the following sentence holds:

$$(x, y) \in r, y \xrightarrow{l} y' \Rightarrow \exists x', x \xrightarrow{l} x' \text{ and } (x', y') \in r$$

The greatest bisimulation relation over S is called *bisimilarity* and shown with $\sim$. This is the traditional definition of bisimilarity. There are many different notions. For this definition, we can say that the symmetric similarity is the bisimilarity. But it is not always true for different notions.

Coalgebraic Bisimulation. For an endofunctor F over a category $\mathbf{C}$, a coalgebra, is a pair (X, α), where X is an object of $\mathbf{C}$ and $\alpha\colon X \to FX$ is a morphism in $\mathbf{C}$. Coalgebras serve as an abstraction of variant transition systems. For example, a labeled transition system $(S, L, \to)$ is a coalgebra (S, γ), where $\gamma\colon S \to \mathcal{P}(L \times S)$ and $\mathcal{P}$ is the powerset functor.

S can be the set of the terms of a programming language given by a signature Σ, and $\to$ can be the set of labeled inductions of the language, given by an operational semantics. Following that, a context C is defined as:

$$C ::= \square \mid f(C, \bar{t}) \mid f(\bar{t}, C) \mid f(\bar{u}, C, \bar{s})$$

$f \in \Sigma$ and by $\bar{t}$, $\bar{s}$ and $\bar{u}$ we mean vectors of terms in S. $\square$ is a placeholder. Assuming $t \in S$, then $C[t] \in S$. We call a relation r congruence iff for terms t and s, and a context C, $t\ r\ s$ gives $C[t]\ r\ C[s]$. A language with its operational semantics is compositional iff the bisimilarity relation over the terms of the language is a congruence.

Howe's method. Howe's method has been traditionally used for compositionality results. *Howe closure* of a relation r is shown by $\hat{r}$ and is defined with the following inference rule:

$$\frac{\bar{t}\ \hat{r}\ \bar{s} \quad f(\bar{s})\ r\ s}{f(\bar{t})\ \hat{r}\ s}$$

Assuming that r is reflexive, then $r \subseteq \hat{r}$, and $\hat{r}$ is a congruence. Additionally, if r is reflexive and symmetric, then $\hat{r}^{\star}$, the transitive closure of $\hat{r}$ is symmetric (the transitive closure trick). So, to prove that bisimilarity is a congruence it is sufficient to prove that $\hat{\sim}$ is a bisimulation. Given the non-symmetric nature of the closure, it is more common to prove that $\hat{\sim}$ is a simulation. It is usually expected from a symmetric simulation to be a bisimulation. But it has not always been the case.

Relators.

References